Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series

Differential and Integral Calculus

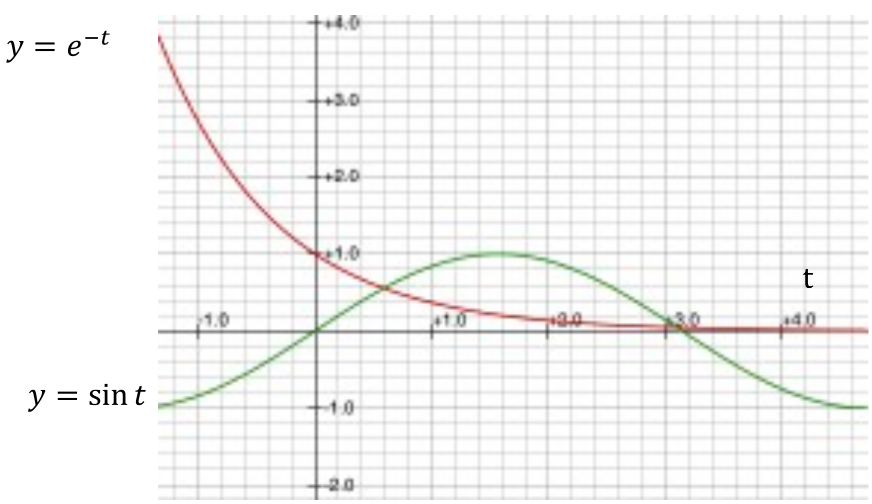
Simple Models Using the Derivative

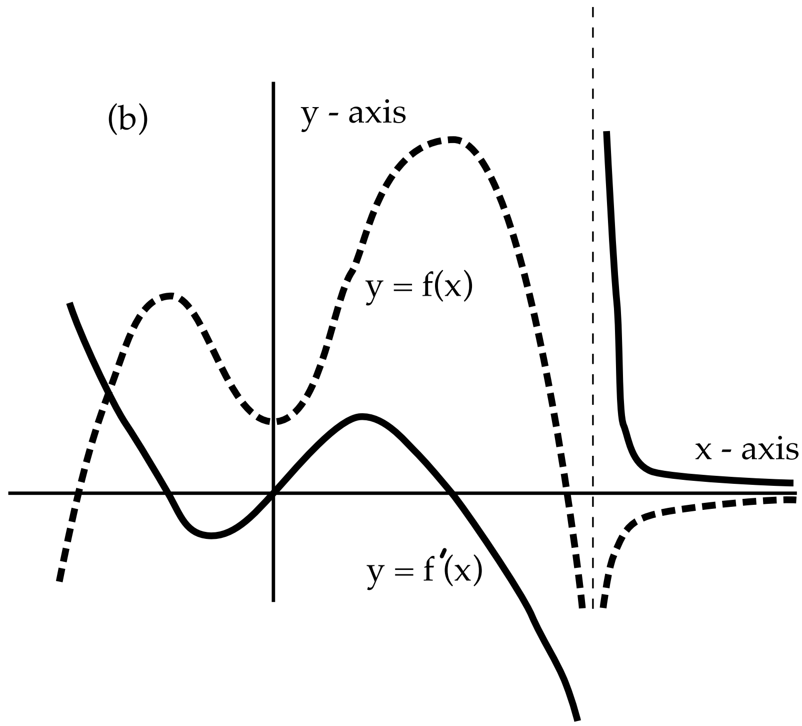
The derivative of a function $f(t)$ is a function $\frac{df(t)}{dt}$.

At a single point $t = a$ the value of the derivative

$\frac{df(t)}{dt}$ is the **rate of change** of f at $t = a$.

This is the instantaneous rate of change – the rate at this one point. However, the value of the derivative depends on the values of f around the point $t = a$.





A simple model

Suppose we are modeling an infection as it spreads through a population.

- P = constant number of people
- $I(t)$ = number of infected people at time t
- So $P - I(t)$ is the number of uninfected people at time t

Our model is that the rate of change of $I(t)$, which is the rate of spread of the infection, is

$$\frac{dI(t)}{dt} = k\, I(t)(P - I(t))$$

This says that the disease is spread with some probability when an infected person meets an uninfected person. These meetings are modeled as constant multiple of the product $I(t)$ and $P - I(t)$. So k is a constant.

This is a differential equation.

It just says, “the rate of change of something equals something”.

Finding a solution is what the technical part of calculus is about.

Understanding what it means is not about technique.

What can we get from this equation?

$$\frac{dI(t)}{dt} = k\, I(t)(P - I(t))$$

The rate of change $\frac{dI(t)}{dt} = 0$ when $I(t) = 0$ or $I(t) = P$. This just says that if there are no infected people then the infected population can’t grow. Also if all people are infected there can’t be any new infections so the rate of change is zero.

Between these values $I(t)(P - I(t) > 0$ so the infected population will be growing.

This is a very simple model and its long-term prediction is that everyone is infected. This is rarely what happens.

We can improve this model – see the video on SIR epidemic models.

Prof. Brian Mortimer
School of Mathematics and Statistics
Carleton University
Ottawa Ontario Canada

brian_mortimer@carleton.ca

March 2012